

methane on these surfaces. It appears from Kemball's researches⁸ that methane permits a similar differentiation between metal surfaces and their capacity to chemisorb gases.

Acknowledgment.—This work has been made possible by a generous grant from Baker and Co. to the James Forrestal Research Center at Princeton which is being used for work in catalysis by

(8) C. Kemball, *Proc. Roy. Soc. (London)*, **A217**, 376 (1953).

platinum metals and for studies in the metal science of these elements. Our thanks are due especially to Mr. Chas. W. Engelhard for financial support, and to Dr. E. F. Rosenblatt and to Dr. W. Cohn for assistance in the preparation and provision of catalytic materials. We express our thanks also to Dr. M. Boudart for supervision and criticism of this work.

PRINCETON, N. J.

[CONTRIBUTION FROM THE FRICK CHEMICAL LABORATORY, PRINCETON UNIVERSITY]

Reduced Equation for Viscoelastic Behavior of Amorphous Polymers in the Transition Region¹

BY A. V. TOBOLSKY AND E. CATSIFF

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A law of corresponding states is proposed for the viscoelastic properties of amorphous polymers in the transition region. Tables are presented from which the modulus-temperature curve (modulus measured after any fixed time t) can be constructed for many polymers.

Introduction

In previous publications,^{2,3} it was proposed that the composite stress-relaxation curves of amorphous polymers in the transition region could be adequately reproduced by the equation

$$\frac{\log E_r(t/K) - \frac{1}{2} \log E_1 E_2}{\frac{1}{2} \log (E_1/E_2)} = -\operatorname{erf} (h \log t/K) \quad (1)$$

where

$E_r(t/K) = E_{r,T}(t)$ = stress/strain in a sample maintained at a constant small strain for a time t at a temp. T

K = characteristic relaxation time, a function of temp. only for a given polymer

E_1 = quasistatic glassy modulus (usually about $10^{10.5}$ dynes/cm.²)

E_2 = quasistatic rubbery modulus (usually between 10^7 and 10^8 dynes/cm.²)

h = a parameter characteristic of each polymer

$\operatorname{erf} x = 2\pi^{-1/2} \int_0^x \exp(-x^2) dx$, the error integral⁴

Furthermore, it was shown that the temperature dependence of K for the polymers studied to date was

$$h \log K_R = f(T_R)$$

with $f(T_R)$ very nearly the same for all polymers. $f(T_R)$ is tabulated in Table I.

$K_R = K/K_d$

$K_d = K$ at T_d

$T_R = T/T_d$

T_d = distinctive temp. which is related to, if not equal to, the glass transition temp.

The Reduced Equation of Viscoelastic Behavior.

—Just as it is very convenient to express compressi-

(1) Part III of a series on elastoviscous properties of amorphous polymers in the transition region.

(2) J. Bischoff, E. Catsiff and A. V. Tobolsky, *THIS JOURNAL*, **74**, 3378 (1952), hereinafter called paper I.

(3) E. Catsiff and A. V. Tobolsky, *J. Appl. Phys.*, in press, hereinafter called paper II.

(4) Tables of the error integral may be found in Jahnke and Emde, "Tables of Functions," B. G. Teubner, Leipzig and Berlin, 1933; J. W. Mellor, "Higher Mathematics for Students of Chemistry and Physics," Longmans, Green and Co., London, 1909.

TABLE I

REDUCED-TEMPERATURE DEPENDENCE OF $f(T_R)$			
T_R	$f(T_R)$	T_R	$f(T_R)$
0.940	1.62	1.000	0.00
.945	1.55	1.005	-.18
.950	1.46	1.010	-.34
.955	1.36	1.015	-.50
.960	1.25	1.020	-.63
.965	1.11	1.025	-.78
.970	0.98	1.030	-.92
.975	.84	1.035	-1.06
.980	.69	1.040	-1.18
.985	.52	1.045	-1.31
.990	.36	1.050	-1.43
.995	.18		

bility data of fluids in terms of reduced temperature, pressure and volume, it would also be very desirable to express the viscoelastic properties of amorphous substances in terms of reduced variables. In this case, the most important reduced variable is the reduced temperature $T_R = T/T_d$. Qualitatively, the viscoelastic properties of amorphous polymers (in the transition region) are very similar at the same value of the reduced temperature. It is the purpose of this paper to establish this relation in a quantitative sense.

Equation 1 is essentially a five-parameter reduced equation for viscoelastic behavior of amorphous polymers in the transition region. However, it is not the simplest reduced equation possible. By comparing the values of h and T_d obtained in papers I and II on six amorphous polymers, it becomes clear that $hT_d = 100 (\pm 3.7)$ (in °K.). Also $\log K_d = -1.45 (\pm 0.12)$ (in hours).

If these values are substituted in equation 1 the following is obtained

$$V = \frac{\log E_{r,T}(t) - \frac{1}{2} \log E_1 E_2}{\frac{1}{2} \log (E_1/E_2)} = -\operatorname{erf} \left[\frac{100}{T_d} (\log t + 1.45) - f(T/T_d) \right] \quad (2)$$

TABLE II
VALUES OF $X = (100/T_d)(\log t + 1.45)$

$T_d, ^\circ\text{C}$ $t, \text{hr.}$	-70	-60	-50	-40	-30	-20	-10	0	10	20
10^{-6}	-2.24	-2.13	-2.04	-1.95	-1.87	-1.80	-1.73	-1.665	-1.605	-1.55
$10^{-5.5}$	-1.99	-1.90	-1.82	-1.74	-1.66	-1.60	-1.54	-1.48	-1.43	-1.38
10^{-5}	-1.745	-1.66	-1.59	-1.52	-1.46	-1.40	-1.35	-1.30	-1.25	-1.21
$10^{-4.5}$	-1.50	-1.43	-1.37	-1.31	-1.25	-1.20	-1.16	-1.12	-1.08	-1.04
10^{-4}	-1.25	-1.195	-1.14	-1.09	-1.05	-1.01	-0.97	-0.93	-0.90	-0.87
$10^{-3.5}$	-1.01	-0.96	-0.92	-0.88	-0.84	-0.81	-0.78	-0.75	-0.725	-0.70
10^{-3}	-0.76	-0.725	-0.695	-0.665	-0.64	-0.61	-0.59	-0.57	-0.55	-0.53
$10^{-2.556}$	-0.545	-0.52	-0.495	-0.47	-0.455	-0.44	-0.42	-0.405	-0.39	-0.38
$10^{-2.5}$	-0.52	-0.49	-0.47	-0.45	-0.43	-0.415	-0.40	-0.38	-0.37	-0.36
0.0100	-0.27	-0.26	-0.25	-0.24	-0.23	-0.22	-0.21	-0.20	-0.19	-0.19
.0316	-0.025	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
.100	.22	.21	.20	.19	.185	.18	.17	.165	.16	.15
.316	.47	.445	.43	.41	.39	.375	.36	.35	.335	.32
1.000	.71	.68	.65	.62	.60	.57	.55	.53	.51	.495
3.162	.96	.91	.87	.835	.80	.77	.74	.715	.69	.665
10.00	1.205	1.15	1.10	1.05	1.01	.97	.93	.895	.865	.835
31.62	1.45	1.38	1.32	1.26	1.21	1.16	1.12	1.08	1.04	1.01
100.0	1.695	1.62	1.55	1.48	1.42	1.36	1.31	1.26	1.22	1.18
316.2	1.94	1.85	1.77	1.69	1.62	1.56	1.50	1.445	1.39	1.35
10^3	2.185	2.08	1.99	1.91	1.83	1.76	1.69	1.63	1.57	1.52
$10^{3.5}$	2.43	2.32	2.22	2.12	2.04	1.955	1.88	1.81	1.75	1.69
10^4	2.68	2.56	2.44	2.34	2.24	2.15	2.07	1.99	1.92	1.86
$10^{4.5}$	2.92	2.79	2.67	2.55	2.44	2.35	2.26	2.18	2.10	2.03
10^5	3.17	3.02	2.89	2.76	2.65	2.545	2.45	2.36	2.28	2.20
$10^{5.5}$	3.42	3.26	3.12	2.98	2.86	2.74	2.64	2.54	2.45	2.37
10^6	3.66	3.49	3.34	3.19	3.06	2.94	2.83	2.72	2.63	2.54
30	40	50	60	70	80	90	100	110	120	130
-1.50	-1.45	-1.41	-1.36	-1.32	-1.29	-1.25	-1.22	-1.19	-1.15	-1.13
-1.34	-1.29	-1.25	-1.21	-1.18	-1.145	-1.11	-1.09	-1.06	-1.03	-1.00
-1.17	-1.13	-1.10	-1.06	-1.03	-1.005	-0.975	-0.95	-0.925	-0.90	-0.88
-1.01	-0.97	-0.94	-0.915	-0.89	-0.86	-0.84	-0.82	-0.795	-0.775	-0.755
-0.84	-0.815	-0.79	-0.765	-0.74	-0.72	-0.70	-0.68	-0.665	-0.65	-0.63
-0.675	-0.655	-0.635	-0.615	-0.60	-0.58	-0.565	-0.55	-0.535	-0.52	-0.51
-0.51	-0.495	-0.48	-0.465	-0.45	-0.44	-0.43	-0.415	-0.40	-0.39	-0.38
-0.365	-0.35	-0.34	-0.33	-0.32	-0.31	-0.305	-0.295	-0.29	-0.28	-0.27
-0.35	-0.335	-0.325	-0.315	-0.305	-0.30	-0.29	-0.28	-0.27	-0.265	-0.26
-0.18	-0.175	-0.17	-0.165	-0.16	-0.16	-0.15	-0.15	-0.14	-0.14	-0.135
-0.02	-0.02	-0.015	-0.015	-0.015	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
.15	.14	.14	.135	.13	.13	.12	.12	.12	.115	.11
.31	.30	.29	.285	.28	.27	.26	.255	.25	.24	.235
.48	.46	.45	.435	.42	.41	.40	.39	.38	.37	.36
.64	.62	.60	.585	.57	.55	.535	.52	.51	.495	.485
.81	.78	.76	.735	.71	.69	.675	.655	.64	.62	.61
.97	.94	.91	.885	.86	.835	.81	.79	.77	.75	.73
1.14	1.10	1.07	1.03	1.00	.975	.95	.925	.90	.87	.855
1.30	1.26	1.22	1.18	1.15	1.12	1.09	1.06	1.03	1.00	0.98
1.47	1.42	1.37	1.33	1.295	1.26	1.22	1.19	1.16	1.13	1.10
1.63	1.58	1.53	1.48	1.44	1.40	1.36	1.33	1.29	1.26	1.23
1.80	1.74	1.68	1.63	1.585	1.54	1.50	1.46	1.42	1.38	1.35
1.96	1.90	1.84	1.78	1.73	1.68	1.64	1.60	1.55	1.51	1.47
2.13	2.06	1.99	1.93	1.875	1.82	1.77	1.73	1.68	1.64	1.60
2.29	2.22	2.15	2.08	2.02	1.97	1.91	1.86	1.81	1.76	1.72
2.46	2.38	2.30	2.23	2.16	2.11	2.05	1.99	1.94	1.89	1.85

Equation 2 is the reduced equation for viscoelastic behavior of the six polymers studied thus far; the relaxation modulus at any time and temperature, $E_{r,T}(t)$, can be obtained from equation 2 and Table I provided that T_d , E_1 and E_2 are known. Since E_1 and E_2 do not differ very much from one polymer to another, the variation of $E_{r,T}(t)$ from one polymer to another should depend mainly on

T_d . From $E_{r,T}(t)$ all other viscoelastic properties can be derived.

The extent to which equation 2 is a valid one for the six polymers studied thus far is shown in Fig. 1. In this figure the new variable Z , related to Y of equation 2 as shown below, was plotted as ordinate on probability paper (e.g., Codex 32,451) in the form

low-temperature rubber at its glass transition is even more striking than the change in properties which conventional thermoplastics undergo on heating.

Determination of T_d from Modulus-Temperature Curves.—In practice, one is likely to be faced with the problem of determining T_d for a given polymer. The modulus-temperature curve is a convenient method for this. If the modulus-temperature curve was obtained at $t_1 = K_d = 127$ seconds, T_d would be identical with T_{50} , the temperature at which $[\log E_{r,T}(t_1) - \log E_2]/(\log E_1 - \log E_2)$ is equal to 50%. T_{50} is easily determined from an enlarged plot of the modulus-temperature curve. Shorter times, e.g., 5 or 10 seconds, are preferable for speedy determination of T_d . For a given T_d , T_{50} is a function of t_1 . We can derive a simple expression for $\Delta = T_{50} - T_d$ for any value of t_1 by noting that when $[\log E_{r,T}(t_1) - \log E_2]/\log E_1 - \log E_2 = 0.50$

$$\frac{100}{T_d} (\log t_1 + 1.45) = f(T_{50}/T_d) = f(1 + \Delta/T_d) \quad (4)$$

In paper I,² we tentatively approximated $f(T/T_d)$ as a linear function of T/T_d , which decreases when T/T_d increases; also $f(T/T_d) = 0$ when $T/T_d = 1$. So we can write $f(1 + \Delta/T_d) = -(\Delta/T_d)g(\Delta/T_d)$, where $g(\Delta/T_d)$, which is given in Table IV, is nearly constant. Making this substitution in equation 3, one obtains

$$\Delta = -\frac{100}{g(\Delta/T_d)} (\log t_1 + 1.45) \quad (5)$$

TABLE IV

VALUES OF $g(\Delta/T_d)$			
Δ/T_d	$g(\Delta/T_d)$	Δ/T_d	$g(\Delta/T_d)$
-0.060	27.0	0.000	36.0
- .055	28.2	.005	36.0
- .050	29.2	.010	34.0
- .045	30.2	.015	33.3
- .040	31.25	.020	31.5
- .035	31.7	.025	31.2
- .030	32.7	.030	30.7
- .025	33.6	.035	30.3
- .020	34.5	.040	29.5
- .015	34.7	.045	29.1
- .010	36.0	.050	28.6
- .005	36.0		

To use equation 4, it is necessary to proceed by trial-and-error. Knowing T_{50} , one assumes a likely value for $g(\Delta/T_d)$, e.g., 36.0, and solves for Δ . This gives a tentative value of T_d , from which a better value of $g(\Delta/T_d)$ can be found, and so on.

If equation 2 describes satisfactorily the viscoelastic behavior of an amorphous polymer, this fact can be conveniently ascertained from an experimental modulus-temperature curve. The procedure would be to determine T_d as described in the preceding paragraph, construct the hypothetical modulus-temperature curve corresponding to this T_d (using Tables II and III), and compare the computed and the experimental values.

Equation 4 shows that the choice of a different time scale causes a shift of the modulus-temperature curve along the temperature axis approximately proportional to \log time. This is another

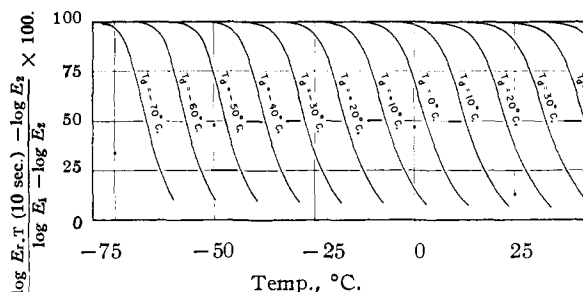


Fig. 2.—Modulus-temperature curves at 10 seconds for amorphous polymers having T_d below room temperature.

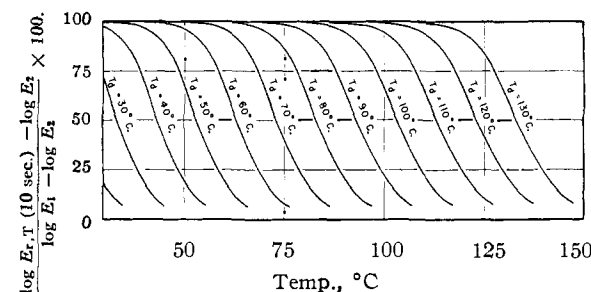


Fig. 3.—Modulus-temperature curves at 10 seconds for amorphous polymers having T_d above room temperature.

manifestation of the time-temperature superposition principle which has proved so fruitful in constructing composite stress-relaxation and dynamic modulus curves. Modulus-temperature curves on the same polymer taken at different times are very nearly parallel.

One difficulty that remains is that of making short-time stress-relaxation determinations to obtain the necessary modulus-temperature information. Many commercially available modulimeters depend on a short-time creep test, e.g., in bending or torsion. Presumably, there exists a simple approximate relation between creep modulus and stress-relaxation modulus.⁵ No adequate experimental proof of this relation in the transition region has been published, however.

Maximum Apparent Heat of Activation for Viscoelastic Behavior.—Another consequence of equation 2 is the existence of a simple relationship between T_d and $(\Delta H_{Act})_{max}$, the maximum apparent activation energy for stress-relaxation. The apparent activation energy is calculated from the Arrhenius equation

$$\Delta H_{Act} = 2.303R \frac{d \log K}{d(1/T)} \quad (5)$$

Making the appropriate substitutions gives (since K_d is constant)

$$\Delta H_{Act} = 2.303RT_d \frac{d \log K_R}{d(1/T_R)} = \frac{2.303RT_d^2}{100} \times \frac{d f(T_R)}{d(1/T_R)} \quad (6)$$

By definition the maximum value of ΔH_{Act} is found when $T_R = 1$, at which point

$$\frac{d f(T_R)}{d(1/T_R)} = -g(\Delta/T_d)_{max} \times \frac{d(T_R - 1)}{d(1/T_R)} = g(\Delta/T_d)_{max} = 36.0 \quad (7)$$

(5) T. Alfrey, Jr., "Mechanical Behavior of High Polymers," Interscience Publishers, Inc., New York, N. Y., 1948, p. 553.

Hence

$$(\Delta H_{\text{Act}})_{\text{max}} = \frac{2.303RT_d^2}{100} (36.0) = 1.65T_d^2 \quad (8)$$

The validity of equation 8 is assessed in Table V, where experimental values of $(\Delta H_{\text{Act}})_{\text{max}}/T_d^2$ have been collected. The data are taken from papers I² and II.³ The average value of $(\Delta H_{\text{Act}})_{\text{max}}/T_d^2$

TABLE V

RELATIONSHIP OF MAXIMUM APPARENT HEAT OF ACTIVATION AND DISTINCTIVE TEMPERATURE

Polymer	$(\Delta H_{\text{Act}})_{\text{max}}$, kcal.	T_d , °K.	$(\Delta H_{\text{Act}})_{\text{max}}/T_d^2$
Polymethyl methacrylate	300	384	2.03
Paracril 26	95.6	241.0	1.64
GR-S	85.0	220	1.75
60/40 Butadiene-styrene	101.9	237.1	1.81
50/50 Butadiene-styrene	99.0	250.8	1.57
30/70 Butadiene-styrene	140.4	285.1	1.73

is 1.74 (± 0.06). This suggests that the maximum value of $g(\Delta/T_d)$ may be slightly greater than 36.0, which is not unreasonable.

Discussion

If the viscoelastic properties of a wide variety of amorphous polymers in the transition region obey equation 2, it is clear that we have developed a kind of law of corresponding states for viscoelastic properties. In particular, the function Y should be identical for all polymers having the same T_d independent of the structure of the polymer. We believe that this will indeed be found valid for many normal amorphous polymers, but will not be true for incompatible copolymers or polyblends.

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PRINCETON, NEW JERSEY

[CONTRIBUTION FROM THE CHEMICAL LABORATORIES OF HARVARD UNIVERSITY]

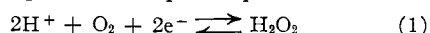
The Polarography and Standard Potential of the Oxygen-Hydrogen Peroxide Couple

BY DAVID M. H. KERN

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The generally accepted equation $O_2 + 2H^+ + 2e^- = H_2O_2$ for the first step in the polarographic reduction of oxygen has been confirmed in the pH range 6–14 by potentiometric null-point measurements with the dropping mercury electrode in buffered hydrogen peroxide-oxygen mixtures. The couple is completely reversible for pH > 11. The standard potential was determined both by the above technique, and by measurements of the oxygen half-wave potential in sodium hydroxide solutions in a cell without liquid junction. The result was $E^0 = +0.695 \pm 0.005$ v. The mechanism of the reduction is briefly discussed.

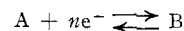
The recent work of Hacobian¹ on the a.c. polarography of oxygen has produced conclusive evidence of the reversibility of the first reduction step in unbuffered neutral and basic solutions. His observations were further confirmed by his discovery of an oxidation wave of hydrogen peroxide in dilute base. Heretofore, although reversible electron exchange between the oxygen molecule and the electrode has at times been postulated,² the over-all process has consistently been stated to be highly irreversible in all media.³ This assumption was supported, at least in the pH range 1–10, by the observation⁴ that the half-wave potential ($E_{1/2}$) of oxygen throughout this range is essentially independent of pH, while according to the accepted equation



for the over-all electrode reaction, reversibility would have led to a shift of 60 mv. per unit change in pH. On the other hand, the standard potential of this couple has been given by Latimer⁵ as +0.682 v. on the basis of data on reaction heats and

entropies, and a simple calculation shows that the overvoltage of the observed oxygen wave (generally placed at -0.05 v. vs. S.C.E.) rapidly diminishes as the solution becomes less acid until it disappears altogether in slightly basic solutions. Furthermore, Berl⁶ has shown that the electrode reaction is reversible at activated carbon electrodes in strongly basic solutions. Practically, the reversibility of the oxygen wave in basic solutions has been exploited because of its steep shape and well developed diffusion plateau, but the appropriate thermodynamic conclusions had never been drawn.

Hacobian found that analysis of the oxygen wave in neutral unbuffered solution gave a linear log plot with a 62 mv. slope, indicating a one-electron process. The log plot analysis derives its validity from the thermodynamic current-voltage equation for the rising part of the wave, and this in turn is based on the assumption that the over-all electrode reaction is given by the expression



where n is determined by the slope of the log plot and A and B are the diffusing species. In one mechanism proposed by Hacobian, B has a coefficient of 3/2, and in the other the reaction involving the one electron is not the over-all process between the diffusing species. Consequently the slope of his plot cannot be used to support either mechanism. An attempt was made to reproduce the log plot of Hacobian in neutral 0.05 M Na₂SO₄

(6) W. Berl, *J. Electrochem. Soc.*, **83**, 253 (1943).

(1) S. Hacobian, *Australian J. Chem.*, **6**, 211 (1953). The same conclusion was very briefly reported by Kalousek, *Collection Czechoslov. Chem. Commun.*, **13**, 105 (1948), on the basis of observations made by a simpler technique.

(2) J. Heyrovsky, "Polarographie," Springer Verlag, Vienna, p. 78.

(3) E.g., I. M. Kolthoff and J. J. Lingane, "Polarography," 2nd Ed., Vol. II, Interscience Publishers, Inc., New York, N. Y., p. 555; M. v. Stackelberg, "Polarographische Arbeitsmethoden," Walter de Gruyter and Co., Berlin, p. 320.

(4) I. Kolthoff and C. Miller, *THIS JOURNAL*, **63**, 1013 (1941).

(5) W. Latimer, "Oxidation Potentials," Prentice-Hall, Inc., New York, N. Y., 2nd ed., 1950, p. 43.